| Centre Number | | | | | | Candidate Number | | | | |
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| For Examiner's Use | |
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| Examiner's Initials | |
| Question | Mark |
| 1 | |
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| TOTAL | |

General Certificate of Education
Advanced Level Examination
June 2012

# Mathematics MFP2

## Unit Further Pure 2

## Thursday 31 May 2012 9.00 am to 10.30 am

**For this paper you must have:**
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Time allowed**
- 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

**1 (a)** Sketch the curve $y = \cosh x$. *(1 mark)*

**(b)** Solve the equation

$$6\cosh^2 x - 7\cosh x - 5 = 0$$

giving your answers in logarithmic form. *(6 marks)*

QUESTION PART REFERENCE

**Answer space for question 1**

QUESTION PART REFERENCE

**Answer space for question 1**

**2 (a)** Draw on the Argand diagram below:

**(i)** the locus of points for which

$$|z - 2 - 3\mathrm{i}| = 2 \qquad \textit{(3 marks)}$$

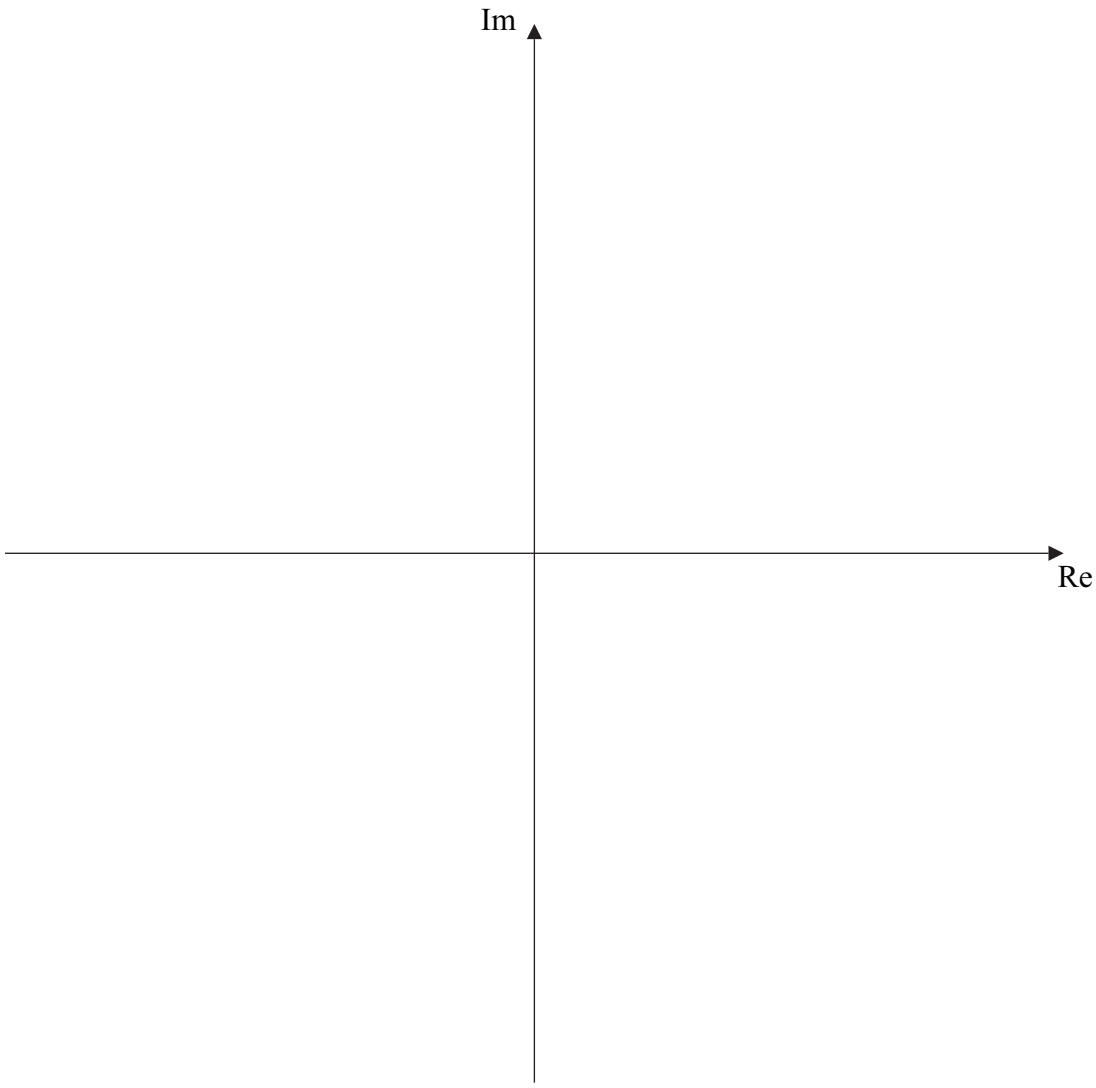
**(ii)** the locus of points for which

$$|z + 2 - \mathrm{i}| = |z - 2| \qquad \textit{(3 marks)}$$

**(b)** Indicate on your diagram the points satisfying both

$$|z - 2 - 3\mathrm{i}| = 2$$

and $$|z + 2 - \mathrm{i}| \leqslant |z - 2| \qquad \textit{(1 mark)}$$



QUESTION PART REFERENCE

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**3 (a)** Show that

$$\frac{2^{r+1}}{r+2} - \frac{2^r}{r+1} = \frac{r2^r}{(r+1)(r+2)}$$ *(3 marks)*

**(b)** Hence find

$$\sum_{r=1}^{30} \frac{r2^r}{(r+1)(r+2)}$$

giving your answer in the form $2^n - 1$, where $n$ is an integer. *(3 marks)*

QUESTION PART REFERENCE

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**4** The cubic equation

$$z^3 + pz + q = 0$$

has roots $\alpha$, $\beta$ and $\gamma$.

**(a) (i)** Write down the value of $\alpha + \beta + \gamma$. *(1 mark)*

**(ii)** Express $\alpha\beta\gamma$ in terms of $q$. *(1 mark)*

**(b)** Show that

$$\alpha^3 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma$$ *(3 marks)*

**(c)** Given that $\alpha = 4 + 7\mathrm{i}$ and that $p$ and $q$ are real, find the values of:

**(i)** $\beta$ and $\gamma$; *(2 marks)*

**(ii)** $p$ and $q$. *(3 marks)*

**(d)** Find a cubic equation with integer coefficients which has roots $\frac{1}{\alpha}$, $\frac{1}{\beta}$ and $\frac{1}{\gamma}$. *(3 marks)*

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**5** The function f, where $f(x) = \sec x$, has domain $0 \leqslant x < \frac{\pi}{2}$ and has inverse function $f^{-1}$, where $f^{-1}(x) = \sec^{-1} x$.

**(a)** Show that

$$\sec^{-1} x = \cos^{-1} \frac{1}{x} \qquad \textit{(2 marks)}$$

**(b)** Hence show that

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{\sqrt{x^4 - x^2}} \qquad \textit{(4 marks)}$$

QUESTION PART REFERENCE

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**6 (a)** Show that

$$\frac{1}{4}(\cosh 4x + 2\cosh 2x + 1) = \cosh^2 x \cosh 2x \qquad \textit{(3 marks)}$$

**(b)** Show that, if $y = \cosh^2 x$, then

$$1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = \cosh^2 2x \qquad \textit{(3 marks)}$$

**(c)** The arc of the curve $y = \cosh^2 x$ between the points where $x = 0$ and $x = \ln 2$ is rotated through $2\pi$ radians about the $x$-axis. Show that the area $S$ of the curved surface formed is given by

$$S = \frac{\pi}{256}(a \ln 2 + b)$$

where $a$ and $b$ are integers. *(7 marks)*

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**7 (a)** Prove by induction that, for all integers $n \geqslant 1$,

$$\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + ... + \frac{2n+1}{n^2(n+1)^2} = 1 - \frac{1}{(n+1)^2} \qquad \textit{(7 marks)}$$

**(b)** Find the smallest integer $n$ for which the sum of the series differs from 1 by less than $10^{-5}$. *(2 marks)*

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**8 (a)** Use De Moivre's Theorem to show that, if $z = \cos\theta + \mathrm{i}\sin\theta$, then

$$z^n + \frac{1}{z^n} = 2\cos n\theta \qquad \textit{(3 marks)}$$

**(b) (i)** Expand $\left(z^2 + \frac{1}{z^2}\right)^4$. *(1 mark)*

**(ii)** Show that

$$\cos^4 2\theta = A\cos 8\theta + B\cos 4\theta + C$$

where $A$, $B$ and $C$ are rational numbers. *(4 marks)*

**(c)** Hence solve the equation

$$8\cos^4 2\theta = \cos 8\theta + 5$$

for $0 \leqslant \theta \leqslant \pi$, giving each solution in the form $k\pi$. *(3 marks)*

**(d)** Show that

$$\int_0^{\frac{\pi}{2}} \cos^4 2\theta \, \mathrm{d}\theta = \frac{3\pi}{16} \qquad \textit{(3 marks)}$$

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**END OF QUESTIONS**

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